LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

FIRST SEMESTER – November 2009

ST 1503/ST 1501 - PROBABILITY AND RANDOM VARIABLES

Date & Time: 12/11/2009 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

SECTION – A

Answer ALL the questions **(10 x 2 = 20 marks)**

1. **Define mutually exclusive events with an example.**
2. **Write down the axiomatic definition of probability.**
3. **If A, B, C are any three events, write down the theoretical expression for the following events: a) Only A occurs b) A and B occurs but C does not.**
4. **Letters are drawn one at a time from a box containing the letters M, A, N, R, O and D. What is the probability that the letters in the order drawn spell the word RANDOM?**
5. **An unbiased die is thrown two independent times. What is the probability that the sum obtained is 8 given that the first throw resulted in an even number?**
6. **If A, B and C are mutually independent events, show that A *U* B and C are also independent.**
7. **A box contains 5 white and 7 black beads. Three beads are drawn in succession. What is the probability that all three are white, if each bead is replaced before the next one is drawn?**
8. **State the Law of total probability.**
9. **Check whether the function defined by $p(x) = (x+2)/5$ for x=1, 2,3,4,5 can serve as a probability distribution of a random variable.**
10. **Define probability generating function.**

SECTION – B

Answer any FIVE questions **(5 x 8 = 40 marks)**

11. **a) For any two events A and B, show that $P(B) \leq P(A)$ if $B \subset A$.** **(4 marks)**

    b) The probability of surviving a certain transplant operation is 0.55.If a patient survives the operation, the probability that his or her body will reject the transplant within a month is 0.20.What is the probability of surviving both of these critical stages? **(4 marks)**

12. **What is a Pascal's triangle? Construct the seventh and eighth rows of Pascal's triangle and write down the expansion of $(x+y)^7$.**

13. **Four groups of children contain respectively 3 girls and 2 boys, 2 girls and 3 boys, 1 girl and 4 boys, 1 boy and 4 girls. One child is selected at random from each group. Find the probability that the four selected consist of 2 girls and 2 boys.**
14. **Arun and Mohan alternately cut a pack of cards and the pack is shuffled after each cut. If Arun starts and the game is continued until one cut a spade, what are their respective chances of first cutting spade?**
15. **Given 'n' independent events A_i, i = 1,2,…, n with respective probabilities of occurrence α_i , find an expression for the probability of occurrence of atleast one of them.**
16. **State and prove Baye's theorem.**
17. **The pdf of the random variable Y is given by**

$$f(y) = \begin{cases} k(y+1), & 2 < y < 4 \\ 0, & \text{otherwise} \end{cases}$$

Find a) k b) P (Y<3.2) c) P (|Y| < 3) d) E(Y)

18. **If X and Y are random variables, show that E (X+Y) = E (X) + E (Y), provided the expectations exist.**

SECTION – C

Answer any TWO questions **(2 x 20 =40 marks)**

19. **a) For 'n' events $A_1,A_2,\ldots,A_n$ show that $P(\bigcap_{1}^{n} A_i) \geq \sum_{1}^{n} P(A_i) - (n-1)$. (10 marks)**

    b) A bag contains 50 tickets numbered 1, 2, 3,…, 50 of which five are drawn at random and arranged in ascending order of magnitude($x_1<x_2<x_3<x_4<x_5$).What is the probability that x_3 =30? (10 marks)

20. **a) Establish addition theorem on probability for any three events. (10 marks)**

    b) A person applies for the post of manager in two firms A and B. He estimates that the probability of his being selected in firm A is 0.75,being rejected in firm B is 0.45 and being rejected in both the firms is 0.55.Find the probability that he will be selected in atleast one of the firms. (10 marks)

21. **a) If X is a random variable with mean μ and variance σ^2 , then for any positive number k, show that P {|X- μ| ≥ k σ}≤ $1/k^2$. (12 marks)**

    b) If Y is a random variable such that E(Y) = 3 and E (Y^2) = 13, use Chebychev's inequality to determine a lower bound for P(-2<X<8). (8 marks)

22. **a) Obtain an expression for the variance of a linear combination of random variables. (10 marks)**

    b) What is the expectation of the number of failures preceding the first success in an infinite series of independent trials with constant probability p of success in each trial? (10 marks)

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